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Jorge Cuéllar[a]; Audelino Alvaro[a]
[a] CHEMICAL ENGINEERING DEPARTMENT, SALAMANCA UNIVERSITY, SALAMANCA, SPAIN

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Fluid–Solid Mass Transfer in Magnetic Filtration

JORGE CUÉLLAR and AUDELINO ALVARO
CHEMICAL ENGINEERING DEPARTMENT
SALAMANCA UNIVERSITY
PLAZA DE LOS CAÍDOS 1-5, 37008 SALAMANCA, SPAIN

ABSTRACT

A theoretical model for the process of magnetic filtration of paramagnetic particles suspended in water and a method for determining the kinetic parameters of the model is proposed. To validate the theory, 10 experimental runs were conducted on a cylindrical filter of 300 mm diameter, containing a concentric iron rod of 250 mm diameter and a bed of spherical iron beads of 8 mm diameter. The suspension was made from iron oxide powder particles in a size range between 2 and 30 μm. The current intensity circulating in the coil which surrounded the filter ranged from 240 to 350 A, and the suspension superficial velocity varied between 1 and 2.2 m/min. The model supposes that the filtration rate is proportional to both the particle concentration in the suspension and the fraction of the iron spheres that are not covered by the deposited paramagnetic particles. The results show that the model can adequately describe the physical phenomena occurring in the filter.

INTRODUCTION

In industry there are many fluids which have to be filtered before, or after, being used. In some cases, filtering is carried out in order to eliminate polluting solids and, in others, in order to recover the useful part of the mixture, whether this be the solid part, the fluid part, or both. Due to the enormous variety of materials that have to be filtered, and the different possible operating conditions, the number of different filtration procedures and apparatus is great.

In the case of filtration of the solids in suspension in a liquid, if the particles are large (>100 μm), separation can be achieved by screening.

For particles between 10 and 100 μm, a variety of separation techniques may be suitable: settling and thickening (which are gravitational methods), cycloning (an inertial route), or through-flow filtration, using a porous medium with openings ranging from 5 to 100 μm. For particles smaller than 10 μm, membranes or cartridges with an adequate pore size are usually employed but, in these cases, the processes only allow the filtration of small qnatities of solids, the filtering medium has to be renewed very frequently, and, often, the solids cannot be recovered from the filters in a form suitable for further processing (1).

In these and other cases, magnetic filters or separators, can be useful (2). This type of filter has an advantage over conventional filters in that the existence of magnetic forces opposing the drag force of the fluid facilitates the retention of the solid particles inside the filter. In order to design these devices, the processes taking place inside them must be understood both qualitatively and quantitatively, and, with this aim, in the case of high gradient magnetic separators or filters, many studies have been carried out on the details of their construction, functioning, and behavior in specific applications (3–9). However, we have found no references to any simple procedure for studying the behavior of filters such that the results obtained allow a change of scale. Hence, the aim of the present investigation is to try to obtain sufficient information on the process of particle deposition in the filtering medium to make it possible to predict the behavior of the filters when the operating conditions vary. More specifically, an attempt has been made to determine the kinetic equation for the filtering of an aqueous suspension of iron oxide particles by means of a magnetic filter. To this purpose a model was proposed for the filtering process, and a kinetic equation derived from it. Subsequently, a series of filtration experiments were carried out in order to determine whether the experimental filtration rate was correctly represented by the theoretical kinetic equation and, if so, to obtain the values of the parameters of the model.

MODEL OF THE FILTRATION PROCESS

In this study the magnetic filtration process consisted of passing an aqueous suspension of paramagnetic particles through a cylindrical container, which on the inside is full of small iron spheres and on the outside has a surrounding coil through which an electric current circulates.

In order to propose a mathematical model of the filtration process, one must first conjecture which of the physical phenomena involved in the process are the most relevant. In this case it can be assumed that the particles in suspension which pass through the filter are subjected mainly to two types of force:

1. The drag force of the fluid. This force is not uniform since it depends on a series of factors which included the interstitial velocity of the fluid, and this varies from zero in the boundary layer surrounding each iron sphere, to a maximum finite value in the center of the spaces between the spheres.
2. The magnetic force acting on the particles, which is not uniform either; the nearer the particle is to the sphere, the greater the force.

Consequently, only those particles which pass close to the spheres (where the attraction force is greatest and the drag force least), or collide with them, will have the maximum possibility of becoming collected. As the whole process is random, the retention rate will be proportional to the concentration of particles circulating in the fluid and to the surface area of the spheres that is useful for retaining particles.

Saturation of the filter will occur when the drag force of the liquid, due to the progressive decrease in the porosity of the filter, increases to such a degree that it surpasses the magnetic force.

Kinetic Equation for Magnetic Filtration

To obtain an equation relating the variation rate of the concentration of particles inside the filter to a variable which will usually be the particle concentration, the following simplifying suppositions are made:

All the surface of the ferromagnetic spheres has the same capacity for retaining the paramagnetic particles in suspension.
Retention of the particles is always carried out by the same mechanism.
The filtration rate will be proportional to the concentration of particles in the liquid and to the surface available for retaining these particles.
If the velocity of the fluid is high enough, the phenomenon is reversible, that is, an equilibrium would be reached.

Taking this into account, the kinetic equation representing the process can be expressed as:

$$-r = -\frac{dC}{dt} = K_f C S - K_d \bar{C} \tag{1}$$

where $-r$ is the retention rate of the particles in the filter, C is the concentration of the substance to be filtered in the liquid phase, t is the time, K_f is the kinetic coefficient of the filtration process (of deposition) of the particles on the surface of the spheres, S is the surface of the spheres adequate for the deposition of particles, K_d is the kinetic coefficient of the detachment of the particles deposited in the spheres, and $\bar{C}$ is the concentration of particles deposited. The meaning of the symbols in the

next equations can be found in the nomenclature section at the end of the paper.

Equation (1) assumes that the net variation or particle concentration in the filter is a function of two terms: a term of particle deposition on the surface of the spheres, which is proportional to the concentration of particles in the liquid and to the surface available for deposition, and, assuming the phenomenon to be reversible, another term of particle detachment, which is proportional to the concentration of particles deposited.

Moreover, we have

$$S = S_0 - K_0\bar{C} \tag{2}$$

and also

$$\bar{C} = K_1(C_0 - C) \tag{3}$$

Then, substituting Eqs. (2) and (3) into Eq. (1) and operating, gives

$$\begin{aligned} -r = -\frac{dC}{dt} &= K_f C(S_0 - K_0\bar{C}) - K_d\bar{C} \\ &= K_f C(S_0 - K_0K_1(C_0 - C)) - K_dK_1(C_0 - C) \\ &= K_fK_0K_1C^2 + (K_fS_0 - K_fK_1K_0C_0 + K_dK_1)C - K_dK_1C_0 \\ &= K_2C^2 + K_3C + K_4 \end{aligned} \tag{4}$$

where

$$K_2 = K_fK_0K_1 \tag{5}$$

$$K_3 = K_fS_0 - K_fK_0K_1C_0 + K_dK_1 \tag{6}$$

$$K_4 = -K_dK_1C_0 \tag{7}$$

Equation (4) shows that, with this model, the net variation rate of the concentration of particles in the suspension inside the filter can be represented by a second-order polynomial in terms of the particle concentration.

In cases where the process is not reversible, $K_d = 0$. Under this circumstance, Eq. (4) can be simplified to

$$-r = -\frac{dC}{dt} = K_2C^2 + K_5C \tag{8}$$

where

$$K_2 = K_fK_0K_1 \tag{9}$$

$$K_5 = K_fS_0 - K_fK_0K_1C_0 \tag{10}$$

In this case the kinetic equation is still a second-order polynomial, but without an independent term.

In this study it was found that, under the experimental operating conditions, filtration was irreversible, since the suspension could be totally filtered, and the experimental filtration rate could be represented by an equation of the type of Eq. (8).

EXPERIMENTAL

Apparatus

Figure 1 shows a diagram of the apparatus used to obtain information on the kinetics of filtration. It fundamentally consists of:

A storage tank for the suspension to be filtered with a capacity of 500 liters; inside there is a impeller for maintaining the suspension homogeneous.

A pump whose motor speed is regulated by a frequency converter which, in turn, is monitored by a computer.

A wedge flow meter with two pressure sensors—connected to the computer—which measure the pressure drop experienced by the fluid on passing through the narrowing.

The filter, consisting of a cylindrical container of 300 mm diameter containing a concentric iron rod of 250 mm diameter and a bed of spherical iron beads of 8 mm diameter and surrounded by an electromagnetic coil which is activated by means of a power supply. The porosity (α) of the bed was 0.4.

A tank for concentrates, a recirculation pipe, and a control computer.

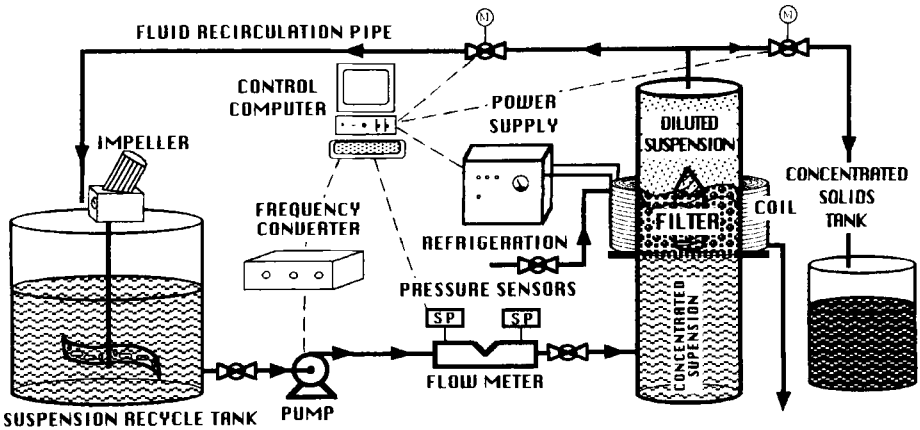

FIG. 1 Schematic diagram of experimental equipment.

TABLE 1
Operation Parameters and Values Obtained for the Kinetic Coefficients

	I									
	240 A			300 A			350 A			
Q	54.5	98.1	118.1	76.4	97.5	118.6	67.8	78.9	96.8	117.5
C_0	1.52	1.66	1.65	1.07	1.73	1.7	1.48	1.61	1.73	1.6
K_F	13,531	23,505	28,168	37,878	38,727	39,046	43,000	43,165	38,727	43,108
K_0	0.0963	0.0976	0.0955	0.109	0.098	0.1	0.098	0.102	0.098	0.109
K_1	0.0046	0.0045	0.0046	0.0053	0.0047	0.0048	0.0047	0.0049	0.0047	0.0048
K_2	5.99	10.3	12.37	21.9	17.84	18.7	19.8	21.6	18	22.6
K_5	3.8	5.3	6.46	12.7	6.1	5.4	11.7	6.6	5.7	4.9

Experimental Procedure

The experiments carried out in order to obtain the filtration rate equation were the following.

First, an aqueous suspension of paramagnetic particles was placed in the storage tank using water and powder particles of Fe_2O_3 in a size range between 2 and 30 μm. Once the suspension was obtained, the flow rate required for the experiment to be performed was fixed by the computer and, with the magnetic field shut down, the pump was connected in order to obtain a stable flow rate, Q, and a homogeneous suspension, of C_0 concentration, throughout the installation. When this was achieved, the magnetic field was connected with a predetermined current intensity I, and since the filtration process commenced from that moment, measurement of the time, t, began. At certain intervals of time, samples were taken of the liquids in the tank and in the pipe leading from the filter, and their composition was analyzed. Filtering continued until the liquid seemed to be totally transparent. Numerous experiments (Table 1) were made varying the flow rate of liquid and value of the magnetic field applied. The current intensity circulating in the coil ranged from 240 to 350 A (15 V), and the suspension superficial velocity varied between 1 and 2.2 m/min.

DETERMINATION OF THE COEFFICIENTS OF THE KINETIC EQUATION OF FILTRATION

From the measurements of the concentration of the suspension at different times, the kinetics of the filtration process were obtained by the following procedure.

Since the filtration rate varies inside the filter according to the variation in the concentration with time and space, a treatment was carried out similar that of a piston flow reactor in unsteady state.

The unsteady-state mass balance applied to a differential volume element of length δx of the filter, as shown in Fig. 2, may be written:

Rate at which mass enters the volume = CQ, g/min
Rate at which mass leaves the volume = $\overline{C}$, g/min
Rate at which mass is deposited (filtrated) in the volume = $(-r)A\delta x$, g/min
Rate at which mass accumulates in the volume = $(dC/dt)A\delta x$, g/min

From which

$$CQ = CQ + Q\frac{dC}{dx}\delta x + (-r)A\delta x + \frac{dC}{dt}A\delta x \tag{11}$$

Simplifying, the balance is

$$\frac{QdC}{A\,dx} + \frac{dC}{dt} + (-r) = 0 \tag{12}$$

Equation (12), resulting from the balance, is a partial differential equation which is not easy to solve analytically. However, we can avoid solving it if we take into account the fact that we are working with a tubular filter in which piston flow could be assumed. In this case, all the fluid elements have the same residence time and there is no mixing of fluid elements with different ages. Under these conditions, each fluid element, during the time that it takes to move from the entry of the filter to the exit, acts as a batch microscopic reactor, or filter, and the equation representing the material balance in each fluid element, i, will be:

Rate at which mass enters the volume = 0
Rate at which mass leaves the volume = 0
Rate at which mass is deposited (filtrated) in a volume = $(-r)V_i$, g/min
Rate at which mass accumulates in the volume = $(dC_i/dt)V_i$, g/min

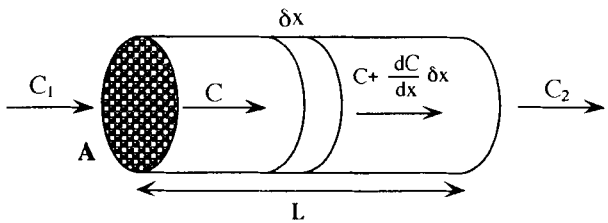

FIG. 2 Diagram showing the filter and a differential volume element.

Thus, from the balance we have

$$-\frac{dC_i}{(-r)} = dt \tag{13}$$

Now, the kinetics can be obtained from Eq. (13) by means of the integration method. By intergrating for a time equal to the residence time, τ, of each fluid element in the filter:

$$-\int_{C(t-\tau)}^{C(t)} \frac{dC_i}{(-r)} = \int_{(t-\tau)}^{(t)} dt = \tau \tag{14}$$

As the residence time is equal to the volume of dead spaces in the filter, V_d, divided by the flow rate, Q, and $(-r)$ can be substituted according to Eq. (8), we can write

$$\tau = \frac{V_r}{Q} = \int_{C_i(t)}^{C_i(t-\tau)} \frac{dC_i}{K_5C_i + K_2C_i^2} = \frac{1}{K_5} \ln\left(\frac{C_i}{K_5 + K_2C_i}\right)\Bigg]_{C_i(t)}^{C_i(t-\tau)}$$

$$= \frac{1}{K_5}\left[\ln\left(\frac{C_i(t-\tau)}{K_5 + K_2C_i(t-\tau)}\right) - \ln\left(\frac{C_i(t)}{K_5 + K_2C_i(t)}\right)\right]$$

and, finally,

$$\tau = \frac{1}{K_5} \ln\left(\frac{C_i(t-\tau)(K_5 + K_2C_i(t))}{C_i(t)(K_5 + K_2C_i(t-\tau))}\right) \tag{15}$$

Subsituting K_2 and K_5, according to Eqs. (9) and (10), in Eq. (15), we have

$$\tau = \frac{1}{K_fS_0 - K_fK_0K_1C_0}$$

$$\ln\left(\frac{C_i(t-\tau)(K_fS_0 - K_fK_0K_1C_0 + K_fK_0K_1C_i(t))}{C_i(t)(K_fS_0 - K_fK_0K_1C_0 + K_fK_0K_1C_i(t-\tau))}\right) \tag{16}$$

Now, Eq. (16) can be used to calculate the value of K_f. To do so, it is necessary to know the value of τ, which can be calculated from the volume of dead spaces of the filter, V_d, which is known, and from the flow rate, Q, fixed for each experiment. The initial concentration value of the suspension, C_0, is measured at the beginning of each experiment, and the value of S_0 can be calculated by taking into account the fact that the particles are spherical with a radius (R) of 0.4 mm and that the density of the iron (ρ) is 7.86×10^6 g/m^3.

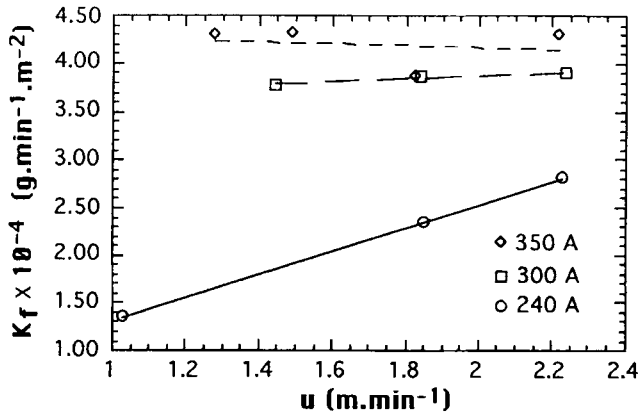

FIG. 3 Variation of the filtration coefficient with the superficial velocity of the fluid.

$$S_0 = \frac{4\pi R^2}{\frac{4}{3}\pi R^3 \rho} = \frac{3}{(4 \times 10^{-4})(7.86 \times 10^6)} = 9.54 \times 10^{-4} \text{ m}^2/\text{g}$$

Finally, the values of the concentrations, C_i, at each time, t, are also obtained experimentally.

With all this information it is possible to calculate the values of K_f, K_0, and K_1 from Eq. (16). Calculation was made by a computer program that determined the K_f, K_0, and K_1 values which gave the least error for the set of equations used. The results obtained are given in Table 1.

Figure 3 shows that there is a linear variation of the value of K_f with the superficial velocity of the fluid in the case of a current intensity of 240 A circulating in the coil. However, when 300 and 350 A pass through the coil, no variations are found in the value of K_f when the velocity of the fluid changes. This suggests that in the two latter cases the magnetic forces are strong enough for the velocity of the fluid not to have any effect on the value of the fluid–solid mass transfer coefficient.

NOMENCLATURE

A	empty cross-sectional area of the filter (m^2)
C	concentration of the substance to be filtered in the liquid phase ($g \cdot L^{-1}$)

C_0 initial particle concentration ($g \cdot L^{-1}$)
$\overline{C}$ concentration of particles deposited [g (of particles)$\cdot g^{-1}$ (of spheres)]
C_i concentration of particles in a fluid element of the suspension to be filtered ($g \cdot L^{-1}$)
I current intensity circulating in the coil (A)
K_d kinetic coefficient of the detachment of the particles deposited in the spheres [g (of spheres)$\cdot L^{-1} \cdot min^{-1}$]
K_f kinetic coefficient of the filtration process (of deposition) of the particles on the surface of the spheres [g (of the spheres)$\cdot$ $min^{-1} \cdot m^{-2}$]
K_0 coefficient of transformation of units in Eq. (1) [$m^2 \cdot g^{-1}$ (of particles)]
K_1 coefficient of transformation of units in Eq. (3) [$L \cdot g^{-1}$ (of spheres)]
K_2 coefficient defined by Eq. (5)
K_3 coefficient defined by Eq. (6)
K_4 coefficient defined by Eq. (7)
K_5 coefficient defined by Eq. (10)
Q Fluid flow rate ($L \cdot min^{-1}$)
$-r$ retention rate of the particles in the filter ($g \cdot min^{-1} \cdot L^{-1}$)
S surface of the spheres adequate for the deposition of particles [$m^2 \cdot g^{-1}$ (of spheres)]
S_0 initial surface of the spheres adequate for the deposition of particles [$m^2 \cdot g^{-1}$ (of spheres)]
t time (min)
V_i volume of a fluid element of the suspension to be filtered (L)
V_d volume of dead space in the filter (L)

Greek Letters

α porosity of the filter
τ residence time of the fluid elements inside the filter (min)

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